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THE REACTION OF THE EYE TO LIGHT

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Communicated and read by Dr C. E. K. MEES, 29th April, 1920.

THE eye is often treated as a physical instrument and as such is analogous almost part for part with a camera. In each case we have a focussing arrangement, a shutter, an iris diaphragm, and a lens directing an image on the sensitive surface. But when the light reaches the sensitive surface our analogy no longer holds. In the case of the camera the light strikes the film or plate and the result is a latent image which may be then developed by any one of several photographic developers. After developement the plate may be studied and the result of the radiant energy units determined. By using the various emulsions and developing under numerous conditions the physico-chemical processes have been rather definitely established. But with the sensitive surface of the eye, called the retina, our determinations are neither so direct nor certain. As in the case of the plate we can measure the intensity of the light entering the eye and approximate rather accurately the flux density at the retina after this light has passed through the various media of the eye. But as to what the action on the retina is we can only speculate and are still more at a loss to interpret the action involved in the resultant brain processes. The light from an object reaches the retina according to the physical laws of light which we know, but the physiological and psychological processes which follow have thus far been approached only by theory. We are conscious of the retinal action of light only indirectly through the resultant perceptions which cannot be measured. Conscious experience is determined by the nature of the stimuli which enter the various sense organs from the world about us, and variations in these stimuli produce variations in consciousness. So to explain thoroughly the visual perceptions we should have to explain the mechanism of the nervous system, as it is this system which serves as a medium for the interaction between an individual and

his environment. Now while we cannot measure sensations and perceptions directly we have just seen that variations in physical stimuli produce variations in mental states, so we can accurately control our physical factors giving rise to various retinal states and determine the various working conditions of the retina. From these results we can then arrive at arbitrary means of measuring perceptions. It has been found that the eye functions over a range of ten billion to one ($10^{10}:1$), that is, the strongest light the eye can tolerate is ten billion times as bright as the faintest perceptible light. The eye remains efficient throughout the greatest part of the range. The eye is thus the widest ranged instrument known and it is as though we had a scale that would weigh diamonds or coal with the same precision. The eye possesses the property of continually renewing its sensitiveness and unless it is exposed to an exceptionally intense stimulation it can continue to function efficiently for hours at a time. When a photographic plate is exposed to light that plate has served its purpose, but with the retina one exposure may be followed

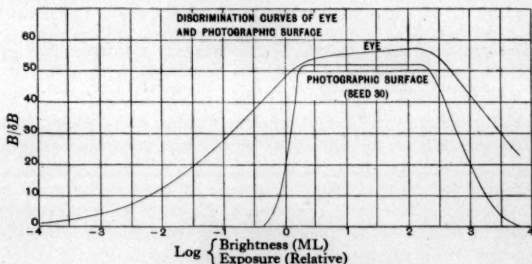


Fig. 1. Curves of the eye and photographic plate.

immediately by another and so on almost without end. Exposing the eye to light is accompanied by a katabolic force which tends to tear down the sensitiveness of the retina, but a counter anabolic force restores this property and the eye continues to function. The retina reaches its greatest sensitiveness after complete dark adaptation, where only the anabolic force is working and all destroyed substance has been replaced. The working range of the best plates produced is very wide, but does not nearly equal that of the eye. It is possible to obtain photographs of very faint stars (with long exposures) and photographic records of very intense sources are taken by stopping down the aperture and supplementing with absorbing filters. An appreciable time interval is necessary in the reception of a stimulus by the retina, but this interval is very small indeed and the visual process may be said to depend largely on the intensity of the stimulus, while in all photographic materials the action is a function of exposure time as much as light intensity. Fig. 1 shows a comparison of the curves of the eye and photographic plate and we see a rather striking resemblance in certain areas of the two curves. The plate

shows a very sudden drop for the lower intensities while the eye gradually slopes to its minimum position. The plotting of logarithms may be misleading to some of us, but it might help if we give the following points of reference: minus 3 represents the average lighted exterior at night, minus 1 the average lighted interior at night, plus 1 interior day and plus 3 exterior day. The gradual slope of the curve for the eye represents the shift from foveal to peripheral vision, and it is this shift that does so much to make the eye such a wide-ranged instrument. By varying the processes in making photographic emulsions, plates may be made fast or slow, sensitive to particular regions of the spectrum and so on through the various characteristics of plates. A plate which is particularly adapted for one type of work, however, is very apt to be unsatisfactory for other purposes, but the normal eye represents a receptor for all types of work. Of course, certain factors enter which control sensitiveness to various functions and we shall now examine some of these factors and the results of laboratory experimentation.

In order to control the factors of visual stimulation an apparatus was built which is known as a Visual Sensitometer. Fig. 2 will give us an idea of the apparatus.

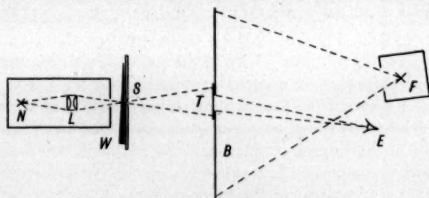


Fig. 2. Diagram of Visual Sensitometer.

In this diagram we see the light from a concentrated source (*N*) focussed through a lens (*L*) on a slit (*S*). The light through the slit illuminates an opal glass window called the test spot (*T*) in the large matt surface which forms the sensitising field (*B*). The intensity by which the test spot is illuminated is controlled by the absorbing wedge (*W*) which operates on metal ways before the slit. At one end this wedge is clear glass and transmits practically all the light and as we move it along it cuts out more and more of the light until the amount transmitted cannot be seen under any condition. By having the wedge accurately calibrated we can determine the brightness of the test spot for all conditions. The flood-lamp (*F*) is used to illuminate the matt surface to any intensity and in this way the observer adapts his eyes to any condition of brightness. It was thought necessary to explain this apparatus rather in detail, as it was used in obtaining most of the data which follow. In most of the experiments the observer was seated directly in front of the test spot so that the distance between the eye and test spot was 35 cm. This distance represents the average reading distance and when kept constant eliminates certain variations which otherwise would enter in case a variable distance were

used. In order to reduce the observers to a standard retinal condition, all of them were kept in total darkness from fifteen to thirty minutes before recording any results, and this treatment counteracts any previous treatment of the retina. Otherwise, an observer who had been working out of doors or in a well-lighted room would give greatly different results from one who had been working in a photographic dark room. The problem of determining the effect of adaptation on the various sensibilities of the retina is one of the first to be studied by means of the Visual Sensitometer. In every-day life we see the workings of this so-called adaptation very often. If we pass from a well-illuminated room to a darker one, or turn off the lights at night, at first we see nothing, but after a few moments we are able to distinguish chairs, tables and other objects and can walk about the room without bumping into the furniture. We could not, however, be able to locate a coin we dropped, regardless of how long we waited. The "getting of our eyes" in this case is strictly an effect of adaptation. In this same example the brighter the lights in the one room and the darker the second room the longer the time necessary to reach a state of adaptation.

As we have previously seen, the retina reaches its highest sensibility when totally adapted to darkness, and in this condition it will be able to perceive the minimal amount of light. The threshold for dark adaptation gives us a determination of the absolute photochemical reactivity of the retina. To determine this value in terms of physical units the test spot was reduced to a 1 millimetre aperture and the observer sat 3 metres from the stimulus. These conditions were then similar to observing a faint star without the disadvantages of atmospheric conditions. An assistant controlled the intensity of the artificial star and recorded the intensity at which it was just perceptible. Numerous readings were taken by three observers over a period of several weeks and the average taken. By taking flashlight photographs of the observers' eyes when fully dark adapted it was possible to determine the diameters of their pupils. Then knowing the intensity and dimensions of the star, the distance to the observer's eye and the pupillary opening we could determine the amount of radiant energy which would just give rise to a sensation. The average from three observers was found to be 19.5×10^{-10} ergs per second, the lowest value being 17.1×10^{-10} . This amount of energy would raise 1 gram of water 1 degree centigrade in 25 years or, eliminating atmospheric conditions, on a totally dark night we could see an ordinary candle at a distance of over 20 miles. In this kind of experiment an observer finds numerous variable disturbing factors and the variations in the separate threshold values are quite marked. The threshold varies with pupillary diameter and this is not a constant. When we are observing a light which is just at the threshold we get decided fluctuations due to attention and fatigue. The circulation of the blood in the small vessels of the retina gives rise to a false light which often interferes with observations. There are also numerous other factors, but probably the greatest single factor is the general physiological condition of the observer from day to day. To test an extreme of this factor one observer took several readings when affected by a severe cold and the threshold was raised so that the results could not be considered at all in obtaining the

average. Just at this point it might be well to mention that the majority of such factors are not subject to control, but are always present to some degree in all visual experiments and the only method in such cases is to take a large number of observations extending over a wide range of time and accept the general average as the threshold value.

Following this determination of the threshold value from a point source of light a series of tests were made to determine the visibility of large and small images of the same total intensity. This was done by varying the distance of the observer from the test spot and also increasing the size of the stimulus. The observer determined the brightness necessary for the star to be detected when the distance was 1.5 metres and 35 cm. and at the latter distance the test spot was varied from a 2 mm. square to a 12 cm. square. Several series were also run for different sized rectangular test spots and the averages determined. When the point source of light is used the radiant energy is concentrated and only a small part of the retina is stimulated, but when the distance between observer and test spot is decreased and the size of spot is increased the amount of the retina affected is larger, that is, the visual angle is increased. The results show that as the visual angle increases the intensity necessary for the perception of the spot decreases, although the actual amount of radiant energy acting on the retina is increased.

While experimenting with the point source of light the question arose as to how much time is required for the light to produce a perceptible sensation. The variations in the threshold values of an observer from one observation to the next made it almost impossible to obtain satisfactory results by timing the action for a given intensity, so carefully calibrated photographic shutters were placed before the test spot of the Visual Sensitometer and the brightness of the spot necessary for it to be seen was determined for exposures ranging from 0.002 second to 4 seconds. It has been shown that, if a stimulus was to be seen at all, in most cases it will be seen within two or three seconds, and exposures greater than 4 seconds almost invariably show a decrease of retinal sensibility. The results obtained show that as the time of exposure is increased the spot brightness decreases, that is, sensibility increases quite rapidly up to 2 seconds. Beyond the latter exposure, various disturbing factors enter and the above-mentioned decrease enters.

If the eye is exposed to a given brightness until fully adapted and this sensitising brightness is then turned off, the least that can be perceived depends upon the length of time in darkness, the intensity of the initial sensitising brightness, and the colour of the light used. Fig. 3 shows the results of adapting our eyes to an initial field brightness, then turning off the field and determining the least perceptible brightness of the test spot after given intervals in darkness. In each of the four initial brightnesses we see a rapid increase in sensibility during the first few seconds followed by a gradual increase up to a period of one hour in total darkness. Some experimenters have reported a continuation of this increase over a period of several hours, but for all practical purposes we may assume an equilibrium has been reached after an hour, and these curves show that the eye is at the same state within an hour for all of the initial brightnesses. When coloured light is used the

growth of adaptation is least and most slowly for red, then come yellow, white, green and blue. If we study the threshold sensibility throughout the brightness range from total darkness to the reflection of full sunlight from white paper, we find that the higher the brightness to which the eye is subjected the higher the least brightness it can detect, so that the sensibility in this case varies inversely with the initial treatment of the eye. When the eye is totally dark adapted, it perceives a 3 cm. square test spot represented by 44×10^{-7} units, while if the adaptation is for a sunny day its minimum is two whole units.

In addition to the differences in adaptation to coloured lights, we find several other interesting facts in colour perception. We might suppose that all radiation

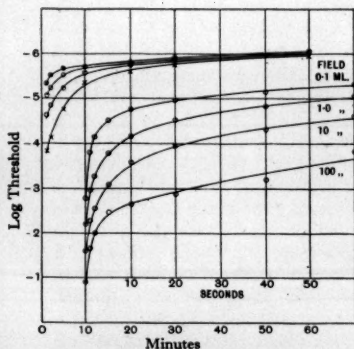


Fig. 3. Time of adaptation curves.

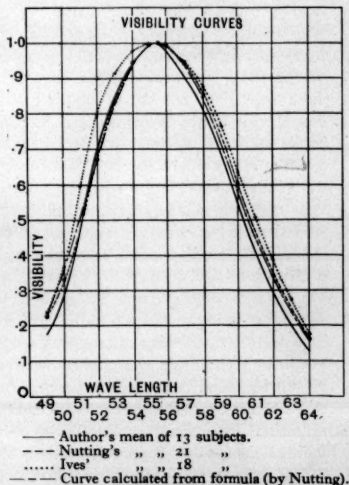


Fig. 4. Visibility of radiation curves.

has the same value as light, but this is far from being true. If we take a standard light source which is kept constant and determine the intensity of various coloured lights necessary to match the standard we find the results as shown in Fig. 4. If we take the least amount of coloured light necessary to match the standard and call that one, then plot the other values on this scale, we get the curve as shown. The maximum visibility, i.e. the region of greatest efficiency, occurs in the green part of the spectrum toward the yellow, and as we pass toward the red and violet ends of the spectrum the efficiency rapidly approaches zero. If we have a green light, slightly yellowish, and wished to change to a red light we would have very

markedly to increase the intensity of the latter to retain the efficiency in our lighted room.

After determining the least we can see under the various conditions, our next problem is to learn the least difference we can detect under the same conditions. That is, if we see a light at a given intensity, how much lighter or darker would it or another light have to be before we could determine they were different. It is due to the fact that objects are of different brightnesses that we can distinguish them from one another, and detail is the name ordinarily given to brightness differences. The ability to see these differences under a wide range of conditions is the fundamental requirement of efficient vision and this ability in the normal eye depends upon the absolute brightness of the objects and their surroundings, the previous condition of the retina, the time of adaptation to the contrasted fields, the area of the fields and their configuration. One method of studying the ability to detect differences in brightness which we will call contrast sensibility is by having two fields side by side. One field is kept at a constant brightness and the other increased or decreased until it is just noticeably different. The two fields may then be made equal again and this operation continued throughout any range of brightness conditions. The other method used was to take a fixed contrast, obtained by covering half the test spot of the Visual Sensitometer with a strip of neutral gray gelatine whose density is known, and determine the brightness necessary before an observer distinguishes between the two fields of the test spot. The effect of length of adaptation to the contrasted fields, the effect of initial sensitising brightnesses, and the effect of size of field were determined for five different contrasts. If we decrease the size of the visual angle by decreasing the size of the test spot we must increase the intensity of illumination in order to perceive the difference within a given time, or increase the adaptation time necessary with a fixed intensity. When the contrast is decreased the same changes are necessary, and if both the contrast and visual angle are decreased then the above changes are greatly increased. The possibilities of application of such results are extensive and apply especially well in camouflage under certain conditions. If we plotted a curve of the contrast sensibility we would see an increase in sensibility as we increase the brightness in the lower intensities. So while we do not see equally well in all lighted places we see practically the same in bright sunshine and a well-lighted study at night. This represents a range of brightness of several thousand to one, and throughout this range the eye is able to detect a difference between two objects of from 1 to $\frac{1}{2}$ per cent.

Another problem in this same line is to determine how bright a light can be tolerated when the eye is adapted to any given brightness. If we are walking at night, automobile lights are quite intolerable, while we should have to take a second look to see if they were burning on a bright day. By means of the Visual Sensitometer it was shown that to a fully dark adapted eye a light of 25 units was judged glaring, but an eye accustomed to a bright sunny day could just tolerate a light of 16,000 units.

When we change the brightness the visual apparatus automatically adjusts

itself, and unless the shift is rather large we are not conscious of the difference and the adaptation is almost instantaneous. In our daily routine the brightness of our environment is ever shifting but, due to the automatic adjustment, we do not notice any changes unless great. It remains to study the process under laboratory conditions and there it is found that an appreciable amount of time is taken in these changes. Very little data have been obtained for the adaptation when changing from lower to higher intensities, although we do know that the change is much more rapid than when the change is from higher to lower. We know that a person "gets his eyes" quicker when turning on a light in a dark room than when darkening a lighted room.

In the process of adaptation and in the determination of the various retinal sensibilities it is seen that there are primarily two factors involved, the retinal process and the change in the size of the pupil. We can only advance theories on the former, but accurate measurements of the pupil have been made. By taking instantaneous photographs of the eyes of an observer throughout the range of brightnesses we are able to determine the size of the pupil under each condition. By placing a small millimetre scale in the plane of the eye and taking large pictures of the eyes the measurements of the pupils were accurate to a tenth of a millimetre. The average maximum diameter of eight observers showed a pupillary diameter of 8 mm. and the minimum was shown to be 2 mm. Although there were some individual differences these values may be taken as the dimensions of the average pupil. By taking moving pictures of the pupil as it closed from a maximum to a minimum and opened from a minimum to a maximum it was possible to get the time and rate of the pupillary reflex. The pupil closes very rapidly and the results show that the average pupil reached its minimum in less than 5 seconds, the greater part of the closing occurring within the first 2 seconds. The rate of opening is quite different, as the average pupil opened to its maximum only after 5 minutes as compared with the same number of seconds to close. This shows us quite definitely that the pupillary process is only a small part in adaptation, as the pupil changes from 2 mm. to 8 mm., a range of 16 to 1, in 5 minutes while the retinal process continues at least an hour and represents a range of at least several million to one. In some experiments it was necessary to use only one eye in making observations, so it was essential to determine the effect of exposing one or both eyes to the sensitising brightness. When one eye is exposed to light and the other eye is closed, the closed eye is really exposed to darkness and the results show that the resultant diameter is a compromise between the size of pupil for a dark adapted eye and an eye exposed to a given brightness. The results of exposing only one eye and both to the several brightness levels previously described show that over the range of ordinary brightness the closing of one eye markedly increases the diameter of the pupil. Eye specialists tell us that when both eyes are normal, no matter what the difference of illumination before the two eyes, the two pupils remain of practically the same size and the result is what is called the consensual light reflex.

Probably one of the most interesting phases of the reaction of the eye to light

is the problem of colour-vision. Most of us can distinguish colours, although occasionally we find people who are either partly or wholly colour-blind and whose light perceptions are merely different shades of gray. We remember that light has a velocity of over 186,000 miles a second and ordinarily light impresses us as being quite simple in nature. If, however, we pass a beam of sunlight through a glass prism we see one phase of the complexity of light. The rainbow effect shows us that white light is made up of a mixture of several colours, and if we study further we learn that each distinguishable colour is made up of a specific rate of light vibrations. If a particular rate predominates we experience a certain colour and if not we have a gray or white sensation. This particular form of vibration may be specified either in terms of the length of the waves or in terms of the number of waves per second. The normal eye is stimulated by light waves whose frequencies

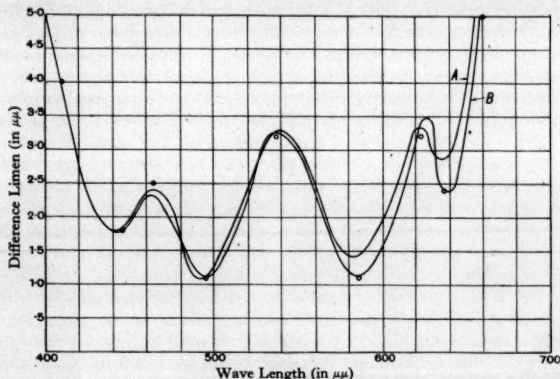


Fig. 5. Hue sensibility.

lie between about 399 and 768 trillion (10^{12}) per second. If 436 trillion waves strike the retina per second a certain impulse is sent along the optic nerve and when it reaches the brain the observer is aware of a certain red. The other method of specifying this certain red would be to state it as $687\mu\mu$ which gives in fractions of a millimetre the wave length. If the frequency is increased to 693 trillion waves a second or the wave length is $432\mu\mu$ then the sensation resulting is a certain violet. Ordinarily, the range of the visible spectrum is from 400 to 700 $\mu\mu$, although under proper laboratory conditions this limit can be extended in each direction. But there are not as many distinguishable colours as might be expected in such a wide range of physical units. If we take a colour in the extreme red of wave length 700 $\mu\mu$ we should have to move toward the blue end of the spectrum 22 units before we noticed a difference in hue, i.e. colour. Wave lengths 700 and 679 $\mu\mu$ would not be distinguishable. If we then move on to 678 $\mu\mu$ we should

notice it to be different. If we now make our standard $678 \mu\mu$ the next perceptible difference would be 13 units, the next 6 and so on we could proceed through the visible spectrum. Fig. 5 shows the result of continuing step by step from 400 to $700 \mu\mu$. In the spectral regions indicated by the lowest points in the curve an average observer is able to detect a change in wave length of a single unit. Other points in the spectrum vary as shown in the curve. In the spectrum, by this method, there were 128 distinct colours between 400 and $700 \mu\mu$. If we add the purples, which are not included in the spectrum, we find about 20 more colours, making in all 148 distinct hues.

In addition to differences in the wave lengths, light waves may also vary in intensity or amplitude of vibration. Intensity of the stimulus determines under given conditions the intensity of the sensation and the resultant mental state is expressed as brightness. That is, if we have a given wave length predominating and the resultant sensation is green we may have a light green or a dark green depending on the brightness of the sensation. So qualities of visual sensation consist of two independently varying groups or series—colour or hue series and the gray series. The gray series includes all shades of brightness between the extremes black and white. It has been found that there are about 700 perceptible shades of gray, but the number of perceptible shades of a colour differs with the colour. It is possible to add a certain gray to a colour and not make that colour lighter or darker, but the change which results gives rise to yet another series sometimes known as the tint series, but also referred to as purity, chroma, and saturation. By starting with a fully saturated colour we can add the proper mixture until we secure a colourless gray. The number of steps between saturation and absence of colour also varies with different colours with a maximum of 25 for any colour. If we combine the number of pure hues with the number of shades and tints for a single hue we get a very large number to express the total number of different visual sensations (over a million). In reality there are probably not over 30,000, as the very light and very dark colours have so many less saturations than the pure spectral colours, and at the extremes, white and black, there are no colours at all. Much work remains to be done in this field of colour research, as this subject has merely been touched.

The purpose of the work called visual sensitometry is to offer new data in the general field of visual research as well as to parallel the tests on photographic materials. If we know the efficiency of the eye under its various working conditions we shall know the accuracy to which we can read photographic results. This information may be applied also in numerous other fields such as illuminating engineering, optical instrument design, ophthalmology and many special branches of physics.

This paper is only a brief *résumé* of some of the experiments carried out in our laboratory, and no attempt is made to compare the results obtained with similar experiments such as those recently reported in England by J. W. French, L. C. Martin, and others.

DISCUSSION

Dr Edridge-Green did not agree with the results obtained in connection with hue perception. He had found that with his instrument an ordinary person could only distinguish about 18 different hues, while a person who could see only about 10 was decidedly colour-blind. The much greater number of hue differences claimed by other workers could, he thought, be explained by the presence of intensity differences and of stray light. He had convinced the late Lord Rayleigh that there were only about 18 steps in the spectrum. By using a double image prism he had projected two portions of the spectrum in a reversed position for the purposes of comparison of hue.

Dr Mees replied that there was no question of instrumental errors in the method used by them, and that intensity variations were corrected for whenever necessary. It was quite an established fact that people can see more than 100 colours.

Mr F. Twyman asked what the function of the iris was if its influence on brightness was small in comparison with the wide range of adaptation of the retina. He said he was not convinced that the method of integrating a curve of measurements of psychological sensitivity differences in order to obtain a sensitivity curve was fundamentally sound.

Dr Mees replied that it was a standard psycho-physical method.

Mr T. Smith considered it was quite clear that the measurements of hue differences carried out at the Kodak Laboratory were those required for very exact physical purposes. He had made experiments some years ago with Dr Edridge-Green's apparatus and had been able, he thought, to distinguish about 22 colours, but it was very difficult to say what the values obtained meant. There had been at that time no arrangement for the juxtaposition of two portions of the spectrum. Apart from the detection of colour blindness, the results obtained were probably of very limited industrial importance; they might, however, be of value in, for example, the dyeing industry, as an indication of the uniformity necessary to give satisfaction, but he thought it had been quite well established that under critical conditions over 100 differences of hue could be distinguished. The late Lord Rayleigh claimed that he could distinguish the hues of the two sodium *D*-lines, and it was unlikely that in making this statement he would fail to consider the effect of intensity differences.

Professor Cheshire said that the two methods of determining hues which were under discussion differed in one very important detail, and that was the sharpness of the boundary between the two surfaces compared. It was well known that in range-finders the attainable accuracy depended very much on the fineness of the separating line of the two partial fields. This fact was very early discovered by Messrs Barr and Stroud, who had devoted a considerable amount of research work to obtaining the sharpest possible line. The arrangement described by Dr Mees obviously satisfied this fundamental requirement, but it was equally obvious that the arrangement described by Dr Edridge-Green did not do so.

Dr R. S. Clay thought the curve of hue differences could be explained by a consideration of the sensitivity curves of the three fundamental colour sensations. The sensitivity maxima would correspond to the positions where these three curves intersected each other, for at the points of intersection the sensitivities corresponding to adjacent sensations are changing most rapidly in opposite directions.

Dr Tomlinson said that he had been associated with Dr Edridge-Green in the work that had been referred to and that the double prism had been used for demonstration purposes at the suggestion of Professor Cheshire. He thought that the method was very coarse as compared with the one employed by Messrs Kodak.

Mr S. A. Willmott suggested that, as double image prisms are made of glass and Iceland spar, instead of Iceland spar only, the two images might not be exactly alike as regards hue, intensity, etc.

Dr E. N. da C. Andrade asked why $700\text{ }\mu\mu$ was taken as the limit at the red end of the spectrum; he could make measurements as far as $770\text{ }\mu\mu$.

He thought that for the purposes of photographically measuring the diameter of the pupil of the eye it was not necessary to use a scale situated near the eye, as the diameter of the cornea could be taken as a reference length.

Mr J. Guild said that the great discrepancy between the hue difference noticeable with Dr Edridge-Green's arrangement, in which the difference to be observed was between opposite sides of a rectangle, and the arrangement described by Dr Mees, in which the two hues were juxtaposed with a very fine dividing line, had a close analogy in the case of brightness discrimination. With an ordinary photometer field, for instance, a difference of about 1 per cent. in the brightness on either side of the dividing line was noticeable; but the brightness at opposite sides of an area may differ by large amounts without being detected. In many telescopes, for instance, the illumination at the edge of the field may only be 50 per cent. or so of that at the centre yet the observer is not conscious of any non-uniformity of the field. The discrimination of a sudden discontinuity in any sensation is always enormously more sensitive than the discrimination between the terminal values of a gradual variation.

Dr Mees had repeated the statement usually made that the pupil size of the two eyes varied simultaneously though the eyes were illuminated differently. In one of his recent papers Mr French had described an experiment in which this was not so. Had this point been tested in the Kodak Laboratory?

He disliked the term "hue sensitivity" as applied to the curves of Jones and others. He would prefer a term such as "frequency sensitivity" or "wave length sensitivity." What the curve really showed was the sensitivity of the eye as a wave-meter at different wave lengths. The sensation of hue, whatever it arose from, was the indicator by which the eye made an approximate measure of wave length, and to speak of the sensitivity of the eye to change of hue was like speaking of the sensitivity of an ammeter to the movement of its pointer.

He asked if the photometric sensitivity for a given brightness of the photometer field varied with the state of dark adaptation of the eye. The values for the Fechner fraction were never stated in relation to this factor

Mr Twyman had mentioned the apparently unimportant effect on brightness which was produced by the variations of the pupil size when compared with the enormous range of retinal adaptation. In his view the function of the pupil variations was not so much to vary the light flux entering the eye, as to cut out the peripheral portions of the lens, and thus reduce the defects of the image due to spherical aberration and reflections from the surfaces of the outer coats of the lens. These defects were most serious at high illuminations. The same function was fulfilled by the pupil variations which accompany extreme accommodation.

Mr Maurice Blood said he was very glad to hear the lecturer deny the often repeated statement that the pupil diameter of one eye was not affected by the light thrown on the other eye; as he (the speaker) had long satisfied himself by direct experiment that in his own case both pupils varied when one eye was exposed to a varying light. Indeed, he had frequently made use of this fact when rifle-shooting, by opening the non-aiming eye, in order to contract the pupil of the aiming eye.

Mr J. Rheinberg said that in Mr French's theory of vision recently propounded to the Society the mechanism of the eye was such that one part dealt with form while another dealt with colour. He thought the distinction was artificial, as he agreed with the view put forward in the present paper that the question of form was merely one of differences of hue and intensity.

Dr Mees, in reply, said that he did not think that the hues of the sodium lines could be distinguished by most observers. Persons who said they could distinguish the hues could generally be tricked by reversing the intensities. He thought that Professor Cheshire's remarks about the fineness of the dividing line were very apposite. The prism they used had to be re-ground six times before a satisfactory dividing line was obtained.

In reply to Dr Andrade he said the limit of $700\text{ }\mu\mu$ had been chosen merely for convenience. The method of using a scale near the eye had ultimately been given up.

He did not think that experiments on the relative sizes of a person's pupils had been carried out in their Laboratory, but he would see that the investigation should be undertaken.

In reply to Mr Guild he stated that the adaptation of the eye did influence the ratio mentioned. He considered that the perception of hue is the same as the perception of wave length.

Mr Reeves communicated the following reply:

The integration of the sensibility function to obtain an expression for the sensation probably originated in the work of Fechner and has been used by Nutting*, Jones† and Blanchard‡. One of the most commonly accepted functions of the pupil of the eye is that of compensating its dioptric effects, but from the stand-

* P. G. Nutting, *Trans. Illum. Eng. Soc.* 11 (1916), 1.

† L. A. Jones, *J. Opt. Soc. of Amer.* 1 (1917), 63.

‡ J. Blanchard, *Phys. Rev.* 11 (1918), 81.

point of evolution the pupillary reflex is largely a matter of protection. It may be considered as a shock absorber which operates automatically to protect the retina from injurious intensities. In every-day experience the eyes are not subjected to very great changes in intensity of light, and the pupillary reflex functions quite satisfactorily. For extreme changes the winking or closing of the lid assists in protecting the eye. Under laboratory conditions the intensity changes may be much greater and more sudden and although the iris functions automatically it is limited by its own physical possibilities.

The question of the consensual reflex is one we hope to investigate in detail in the near future. In the general practice and in the literature of Ophthalmology the statement made in the paper seems to be generally accepted. At least, it is simple to demonstrate that changing the light entering one pupil affects the diameter of the other pupil. If a person watches the pupil of his right eye by the aid of a mirror and then alternately covers and uncovers his left eye he will see the right pupil dilate and contract.

The criticism of the term "hue sensitivity" as used is well founded and calls for further thought on this subject. Regarding the ability of an observer to distinguish the hues of the sodium lines the problem is masked by the difference in intensity between the lines. Some trained observers have told me they could undoubtedly perceive the difference in hue, irrespective of intensity. Practice in such observations is quite a factor and it is a problem for investigation when the intensities are made equal and judgments are made on hue differences alone. At this time we are building an apparatus in the Kodak Laboratory which will be well adapted to this problem.

ON THE FIELD OF VIEW OF A GALILEAN TELESCOPE

By H. A. HUGHES, F.O.S.,
AND P. F. EVERITT, B.Sc., F.R.A.S., F.INST.P., F.O.S.

Read and discussed, 14th October, 1920.

OF recent years there has been a re-awakening of interest in the Galilean telescope with the natural consequence that the problem of enlarging its field of view has again received attention. Unfortunately the theory of the subject as developed in various text-books has not been treated in the simplest manner and the purpose of this short paper is to present a method of considering the subject in such a way as may, we hope, clear away a good deal of the fog at this moment surrounding it.

In every telescopic system we have an entrance pupil and an exit pupil which are so related to each other that the exit pupil is the image of the entrance pupil, and, if the magnification of the telescope be m , the exit pupil will be $\frac{1}{m}$ times the size of the entrance pupil. In the first place we will suppose the telescope to consist of two thin lenses, a positive of focal length f_1 forming the objective, and a negative of focal length f_2 (numerical) which will form the ocular. They will of course be separated by the difference of their focal lengths, to give a telescopic system of magnification m . The observer's eye will be placed behind the ocular and the distance of the centre of rotation of the eye from the ocular will be denoted by d . The entrance pupil will be the aperture of the objective and its semi-diameter will be called P . The exit pupil is then of semi-diameter $\frac{P}{m}$ and is situated a distance

$\frac{m-1}{m} f_2$ in front of the ocular. It is clear that the pupil of the observer's eye cannot be brought into this position, consequently the exit pupil will operate as a porthole bounding the field of view; further, since the observer's eye is accommodated for telescopic vision and the porthole is only a short distance in front of the eye, the porthole will not be sharply defined.

When it is desired to search the field of view of a telescope, the eye is rotated to bring the image of each part of the field in turn on the fovea centralis, and therefore the centre of projection to determine the field of view of the Galilean telescope will be, not the pupil of the observer's eye, but its centre of rotation, which lies at a distance d behind the ocular. Returning to the exit pupil of semi-diameter $\frac{P}{m}$, its distance from the centre of rotation of the eye is $\frac{m-1}{m} f_2 + d$ and

it will consequently subtend a semi-angle whose tangent is $\frac{P}{m}$ divided by $\frac{m-1}{m} f_2 + d$.

Hitherto it has been tacitly assumed that the diameter of the pencil of parallel rays entering the eye through the telescope from any object has been infinitely narrow. If the pupil of the eye has a semi-diameter p , and the eye be directed at the edge of the porthole, it is obvious that only about half the pupil of the eye will be receiving light from the object; to fill completely the pupil of the eye, the eye must be directed at a point at a distance p inside the porthole edge. Similarly, when directed at a point distant p outside the porthole edge, the eye will just fail to receive any light at all. We have thus found three fields of view: the field of maximum illumination for which the whole of the eye pupil is filled with light, the extreme field lying inside the boundary at which the eye pupil just fails to receive any light, and the field whose boundary lies between these and for which the eye pupil is half filled with light. We may write these in mathematical form. The field of view of maximum illumination

$$= 2 \tan^{-1} \frac{\frac{P}{m} - p}{\frac{m-1}{m} f_2 + d} = 2 \tan^{-1} \frac{P - mp}{(m-1)f_2 + md}.$$

Similarly the total field of view

$$= 2 \tan^{-1} \frac{P + mp}{(m-1)f_2 + md},$$

and the so-called mean field of view

$$= 2 \tan^{-1} \frac{P}{(m-1)f_2 + md}.$$

These formulae are necessarily the same as those arrived at in the usual text-books; the term $(m-1)f_2$, which is the distance between the objective and ocular, might have been called l the length of the telescope, but it will be seen presently that its present form is the more useful one when considering by what means the field of view may be enlarged.

In practice, the field of view outside the so-called mean field may be disregarded, as the illumination is less than half of that at the centre. It will be found convenient to call the field of maximum intensity the "central" field and the outer region up to the limit of the mean field the "outer" field. In the central field, since the pupil of the observer's eye is completely filled with light, the brightness of the image is the same as that of the object viewed directly (less the small percentage due to reflections and absorption), and this is true irrespective of the magnification or diameter of the objective. Thus the firm belief of so many users of Galilean binoculars, that larger objectives will give a brighter image, is seen to be contrary to fact. The effect of increasing the diameter of the objective (i.e. increasing P) is to increase the size of the central field, not its brightness; the diameter of the mean field is also increased by the same amount. Unfortunately there are limits to the advantage so to be gained in binoculars, owing to the inter-pupillary distance restricting the diameter of the objective to a little over 2 inches.

The next factor available is f_2 , the focal length of the ocular; it is at once obvious that by decreasing the value of f_2 both the central and mean fields will be enlarged

in the same proportion. This is rather interesting as it shows that it is possible to construct two different Galilean binoculars having the same magnification and central field, but one having a larger mean field than the other, according to whether the size of the central field is obtained by decreasing f_2 or increasing P . The gain in field that may be obtained in this manner is limited by the difficulty of correcting the aberrations sufficiently well when f_2 is reduced much below 1 inch.

There are still two more factors d and m ; d , the distance of the centre of rotation of the observer's eye from the ocular, is not under the control of the optical designer, although it is an important factor. It serves to remind us how important it is to get the eyes as close to the oculars as possible; further, an observer wearing spectacles or having very deeply sunk eyes will not get such a large field of view as another not wearing spectacles or having prominent eyes.

The magnification m remains to be considered; this is generally specified beforehand, so that it is not usually possible to vary it in any way. The effect of any alteration in m is of a twofold nature, as it affects the central field and the mean field rather differently. The effect of increasing m is to decrease the mean field, but the effect on the central field is more serious, as it is decreased still more than the mean field, owing to the increasing value of the subtractive term mp in the numerator. The proportion of the central field to the mean field is thus reduced by increasing the magnification.

The semi-diameter of the eye pupil, p , is also not under control, although it varies considerably from mid-day to dusk. The effect produced is to restrict the area of the central field as the pupil enlarges, but there is no effect on the mean field. Obviously the effect is more serious the larger the magnification m .

Hitherto only "thin" lenses have been considered. It is now necessary to examine the effect of utilising lenses (as must be done in all actual constructions) of finite thickness. In the first place the ocular will not have a central thickness much exceeding 1 to 1½ mm., the effect of which may be disregarded unless the lens be of a meniscus form. In that case the principal points will lie on the side with the deeper curve, which will be towards the eye, but the gain so obtained except for very low magnifications will be about neutralised by the observer's inability to get his eye quite as close as before to the ocular, provided the lens is reasonably thin. If the ocular were made in the form of a very thick meniscus, it is conceivable that some advantage could be obtained which would be more pronounced as the magnifying power is increased. Considering the objective, with suitable thickness for actual constructions, the entrance pupil still remains the front edge of the objective cell and the position of the image of this formed now by the objective and ocular is practically the same as in the case of the thin lens; the effect is small and cannot well be used as a factor in designing, although in nearly every case of a doublet objective it will help the field a little. There remains the possibility of using for either objective or ocular compound systems containing considerable separations. Taking the ocular first, it is possible to have either two negative lenses with a separation or a front negative lens with a positive next the eye; in the first case the principal points will lie inside and the effect is to increase artificially the

distance from the second principal point to the centre of rotation of the eye, so that field is lost; in the second case, although the effective distance to the centre of rotation of the eye is decreased, the front negative lens, if made of existing optical materials, will of necessity have such a small diameter that it will act as a field stop and there will be no gain but a probable loss.

There remains the case of the objective consisting of two components with a separation. If both components are positive, the edge of the front lens cell will be the entrance pupil and will lie in front of the first principal point so that its image through the complete objective will lie in front of the second principal point and there will be a loss, not gain, of field. This is the construction given by Mr H. S. Ryland*. It is not suggested that he did not get an increase of field, but the cause must be looked for in the short focus of the ocular (18 mm.) and not in the additional positive lens which should be regarded as forming the second member of a compound objective system. An alternative construction of a front positive lens with a negative at a suitable distance behind it really amounts to using an ocular consisting of two separated negatives, and as already shown, this is inferior, as regards the field of view obtainable, to a single ocular of the same focal length as the combination.

Description of Exhibits.

By the courtesy of Messrs Henry Hughes & Son, Ltd., the following examples of Galilean constructions were exhibited to illustrate the application of the general theory.

The "Liverpool." A day and night binocular of power $4\times$ and field $4\frac{1}{2}^\circ$, having object glasses of $2\frac{1}{8}$ inches aperture. This glass has fixed interocular distance and may be regarded as the older standard type of marine binocular.

The "Pilot." A low power night glass of power $2\frac{1}{2}\times$ and field varying from $8\frac{1}{2}^\circ$ to 10° according to how close the observer's eye is placed with respect to the ocular. The object glasses have 2 inches diameter and fixed interocular distance. The field has been obtained by reducing the value of f_2 , thus producing a short glass which is very convenient for the pocket.

The "Bridge." A day and night glass similar to the "Liverpool" in type and having power $4\times$ with a field of $4\frac{1}{2}^\circ$ to 5° . The objectives are, however, only $1\frac{3}{8}$ inches in diameter to enable a good range of interocular distance to be given by the usual bending bar construction. Although this glass is much smaller than the "Liverpool," it has the same magnification, together with a rather larger field. The dimensions are so reduced that this glass will conveniently go into the pocket.

Galilean Monocular. This glass has been designed for use in sextants and gives a power of $2\times$ with over 12° of field. The object glass aperture is 1.6 inches, so that the length becomes very short indeed, as will be seen when the above data are inserted in the formulæ given and the focal lengths ascertained.

* H. S. Ryland, *Trans. Opt. Soc.* 19 (1918), 101.

DISCUSSION

Mr H. S. Ryland said that in the design of the Galilean binocular which he (the speaker) had described to the Society, the chief aim was to increase the aperture ratio of the objective. In the sample instruments constructed the extreme limit had been reached.

Instructor-Commander T. Y. Baker suggested that it would have been more useful to put the formula for the main field in terms of the aperture ratio of the objective.

He pointed out that extra field could be obtained by moving the eyes slightly off the optical axes of the binocular.

The author, in reply, said that he had obtained the same result as Mr Ryland, but with less loss of light.

He agreed that the field of view could be artificially increased by moving the eyes or by making the interocular distance somewhat smaller than the correct value, but he thought that such a procedure was a misuse of the instrument and one not to be recommended.

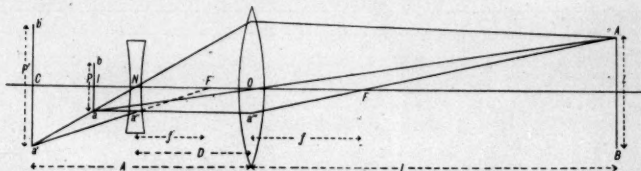
THE USE OF TELESCOPES WITH INTERNAL FOCUSSING FOR STADIA SURVEYING

BY MAJOR E. O. HENRICI, R.E.

MS. received, 25th October, 1920. Read and discussed, 11th November, 1920.

TELESCOPES with an internal negative focussing lens, with cross hairs rigidly fixed in the tube carrying the object glass, have been introduced of late years for use with surveying instruments. Such telescopes have several advantages; they can be made dust and damp tight, or even water tight, and the collimation is less liable to variation during focussing. When an ordinary draw tube is used for the focussing it is difficult to make and keep the movement absolutely true so that the cross hairs remain on the optical axis of the object glass when the tube is moved. With the internal focussing lens the cross hairs do not move, and not only can the moving part (*viz.* the negative lens) be more easily guided, so that its excentric movement is small, but also a given excentric movement has much less displacing effect on the image, relative to the cross hairs, than an equal movement of the cross hairs themselves. This point has been dealt with in the *Zeitschrift für Instrumentenkunde* for 1909, p. 329.

Such a telescope is equivalent to one having an object glass of varying focal length, and the angle subtended by the stadia hairs in the diaphragm will vary according to the focal length, *i.e.* according to the distance of the object on which the telescope is focussed. In the ordinary telescope $L = kl + f + c$, where L is the distance from the vertical axis of the instrument to the stadia rod, l the intercept on the rod between the stadia hairs, f the focal length of the object glass, c the distance of the object glass in front of the vertical axis, and k a constant. In the case under consideration neither k nor f are constants, and it may be of interest to investigate the practical effect of this in stadia working.



In the figure AB (length l) is the portion of the stadia rod viewed between the cross hairs $a'b'$. O is the object glass with focal length f and first focal point at F , and N is a negative lens with focal length f' and first focal point at F' . A is the fixed

distance between the cross hairs $a'b'$ and the object glass O , L the distance from the staff to the object glass, and D the distance between the lenses when an image of AB is formed at $a'b'$.

If the lens N were not present an image of AB would be formed at ab (length of image p), distant f_2 behind the object glass, and a ray from A passing through F would take the path $Afa'''a''a'$, the portion $a'''a$ being parallel to the axis, hence

$$\frac{AB}{ab} = \frac{L-f}{f}, \text{ or } L = \frac{fl}{ab} + f = \frac{fl}{p} + f = kl + f,$$

where k is a constant; this is the ordinary formula for stadia work. With the negative lens in position the ray $Afa'''a''$ will be refracted at a'' to a' where $F'a''a'$ is a straight line, hence

$$\frac{p'}{p} = \frac{CF'}{NF'} = \frac{f' + A - D}{f'} \text{ and } L = \frac{f(f' + A - D)}{f'p'} l + f.$$

Here A , f , and f' are constants, but D increases as L diminishes.

Let $D = D_\infty + d$ where D_∞ is the value of D when the telescope is at stellar focus; D_∞ is then a constant, and

$$L = kl - \frac{fld}{f'p'} + f, \text{ where } k = \frac{f(f' + A - D_\infty)}{f'p'}. \dots (1).$$

It remains to find how the second term varies as L varies. If we simply say that for practical purposes $L = kl$, then strictly the distance L is not measured from the object glass but from a point distant $E = f - \frac{fld}{f'p'}$ in front of the object glass. The problem resolves itself into finding the values of E corresponding to various values of L . To express E in terms of L leads to a rather complicated formula, but it is comparatively simple to express both E and L in terms of d . From the figure

$$\frac{A - D}{f_2 - D} = \frac{p'}{p} = \frac{f' + A - D}{f'}.$$

Hence
$$f_2 = \frac{-D^2 + DA + f'A}{f' + A - D}, \text{ also } L = \frac{ff_2}{f' - f}.$$

Put $D_\infty = nA$ ($n < 1$), then $D = nA + d$, from which

$$f = \frac{A^2n(1-n) + f'A}{f' + A(1-n)}, \text{ or } f' = \frac{A(1-n)(f - nA)}{A - f}.$$

Also
$$f' + A - D_\infty = \frac{A^2(1-n)^2}{A - f}.$$

Hence
$$f_2 = \frac{-d^2(A-f) + Ad(1-2n)(A-f) + A^2(1-n)^2f}{A^2(1-n)^2 - d(A-f)},$$

and
$$L = f \cdot \frac{-d^2(A-f) + Ad(1-2n)(A-f) + A^2(1-n)^2f}{-d^2(A-f) + d\{A^2(1-2n) + f(2nA-f)\}}.$$

From (1)
$$kl \left(1 - \frac{d}{f' + A - D\infty} \right) = L - f,$$

hence
$$kl = \frac{(L - f) A^2 (1 - n)^2}{A^2 (1 - n)^2 - d(A - f)} = \frac{f^2 A^2 (1 - n)^2}{-d^2 (A - f) + d \{ A^2 (1 - 2n) + f(2nA - f) \}},$$

and
$$E = L - kl = \frac{f \{ A(1 - 2n) - d \}}{A(1 - 2n) + f - d}.$$

If $n = \frac{1}{2}$, as indicated in Zeiss' British patent No. 25622 of 1910, then E reduces to $\frac{-fd}{f-d}$, which is zero when the telescope is at stellar focus. If $n < \frac{1}{2}$, E is positive, i.e. the apex of the tacheometric angle, or the point from which L is measured, is thrown forward, but if $n > \frac{1}{2}$, E becomes negative and the apex is thrown back towards the eyepiece. Table I shows the values of E and f' when $A = 1$ for various values of n , and for $f = .80, .85$ and $.90$. Table II shows the values of d , L and E for three telescopes for which (1) $f = .9, n = .4, f' = 3.0$, (2) $f = .9, n = .5, f' = 2.0$, and (3) $f = .9, n = .6, f' = 1.2$, all distances being measured in terms of the length (A) of the telescope. Table III gives corresponding values for similar telescopes, except that $f = .8$. For mechanical reasons it is inconvenient to have D greater than $.75 A$, and the values greater than this have been shown in italics.

If R is the distance from the centre of the instrument to the staff, and c the distance from the centre of the instrument to the object glass, then $R = kl + c + E$. It will be seen from the tables that $c + E$ decreases for any given range as n increases, but that its variation with range for a given telescope increases as n increases. The use of a short focus negative lens (high value for n) in order to reduce the absolute value of $c + E$ will introduce complications into the optical corrections of the lens system, and impair the optical properties of the telescope.

With a 12-inch telescope which follows the Zeiss patent ($n = \frac{1}{2}$), E varies from 0 at long ranges to about $-.1$ ft. at 28 ft. and $-.25$ ft. at 15 ft. The vertical axis of the instrument will be about 6 inches behind the object glass, so that if we simply take the range from the instrument as kl the error will vary from about 6 inches at long ranges to 5 inches at 30 ft. and 3 inches at 15 ft. In view of the fact that such short ranges will seldom be measured, and that at long ranges the errors of reading the staff will be greater than 6 inches, the error in assuming that the telescope is anallatic is generally negligible. If a shorter telescope be used the errors will be smaller. It must be remembered that in plotting detail an error of a foot in the distance measured is in most cases of no consequence, as it cannot be plotted except on very large scales. In running traverses short sights will seldom be taken and with long sights the error is smaller than that due to reading the staff. If a closer approximation is required it will in almost all cases suffice to add a constant equal to the distance between the axis of the instrument and the object glass, and the residual error will not exceed 2 or 3 inches at any distance. If this is of importance (it can only be of importance where great accuracy is required at very short ranges) the distances corresponding to the various stadia readings should be tested and tabulated for each instrument in use, but it is very seldom that this will be worth while.

Table I. Values of E and f' for $A = 1$.

n	$f = .80$		$f = .85$		$f = .90$	
	E	f'	E	f'	E	f'
.30	+ .267	1.70	+ .272	2.57	+ .277	4.20
.35	+ .218	1.46	+ .232	2.17	+ .225	3.58
.40	+ .160	1.20	+ .162	1.80	+ .163	3.00
.45	+ .089	.96	+ .089	1.47	+ .090	2.47
.50	$\pm .000$	.75	$\pm .000$	1.33	$\pm .000$	2.00
.55	- .114	.56	- .113	.90	- .112	1.57
.60	- .266	.40	- .262	.67	- .256	1.20
.65	- .480	.26	- .464	.47	- .450	.88
.70	- .800	.15	- .755	.30	- .720	.60

Table II. Values of L and E for $f = .9$, $A = 1$.

d	$n = .4, f' = 3.0$		$n = .5, f' = 2.0$		$n = .6, f' = 1.2$	
	L	E	L	E	L	E
.000	∞	+ .163	∞	$\pm .000$	∞	- .257
.005	531	+ .160	453	- .005	370	- .266
.01	268	+ .157	228	- .010	157	- .274
.02	135	+ .150	116	- .020	95	- .291
.03	91	+ .143	79	- .031	64	- .309
.04	69	+ .136	59	- .042	49	- .327
.05	55.5	+ .129	48	- .053	39.5	- .346
.06	47	+ .121	40	- .064	33.5	- .365
.07	40.5	+ .114	34	- .076	29	- .386
.08	36	+ .106	31	- .088	26	- .407
.09	32	+ .098	28	- .100	23	- .428
.10	29	+ .090	25.5	- .112	21	- .450
.15	20.5	+ .047	18	- .180	15	- .572
.20	16	$\pm .000$	15	- .257	12	- .720
.25	14	- .053	12	- .720		

Table III. Values of L and E for $f = .8$, $A = 1$.

d	$n = .4, f' = 1.2$		$n = .5, f' = .75$		$n = .6, f' = .4$	
	L	E	L	E	L	E
.000	∞	+ .160	∞	$\pm .000$	∞	- .267
.005	231	+ .157	200	- .005	170	- .276
.01	116	+ .153	100	- .010	86.5	- .284
.02	59	+ .147	52	- .020	44	- .303
.03	40	+ .140	33.5	- .031	29.5	- .323
.04	30	+ .133	26	- .042	22.5	- .343
.05	24.5	+ .126	21	- .053	18	- .364
.06	20.5	+ .119	18	- .065	15.5	- .386
.07	18	+ .112	15.5	- .077	13.5	- .407
.08	16	+ .104	14	- .089	12	- .431
.09	14	+ .097	12.5	- .101	11	- .454
.10	13	+ .089	11	- .114	10	- .480
.15	9	+ .047	8	- .185	7	- .622
.20	7	$\pm .000$	6.5	- .266	5.5	- .800
.25	6.5	- .053	5.5	- .364	4.7	- 1.029

DISCUSSION

Mr W. Shackleton described a series of experiments with a level in which distances derived from stadia measures were compared with actual measured values of distance. The level had internal focussing with a negative lens, and when focussed for infinity the stadia interval gave a reading correct to 1 per cent.

Actual distance	Stadia measure	Error per cent.
10 ft.	9.78 ft.	2
20	19.72	1.5
30	29.76	1.0
40	39.70	1.0
50	49.83	.4
60	59.71	.5
100	99.74	.25
Mean x	.75	

He was glad to see Major Henrici's theoretical account of the subject.

Instructor-Commander T. Y. Baker pointed out that if .25 ft. is added to all the results given by Mr Shackleton the general percentage error would be decreased.

Professor Cheshire suggested that possibly better average results could be secured by making the readings correct for some mean distance rather than for infinity.

Mr R. W. Cheshire thought that Mr Shackleton's values agreed very well with those determined theoretically.

(Communicated remarks)

The approximate data of the Zeiss Level No. 1, as made by Messrs T. Cooke & Sons, of York, are as follows:

Focal length of object glass, 5.8".

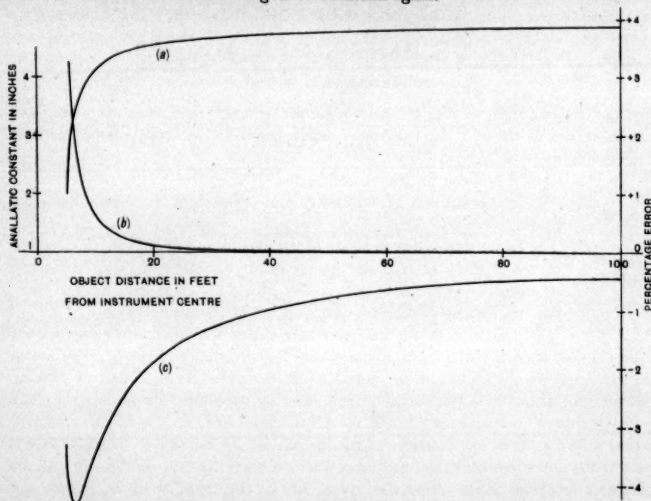
Focal length of negative focussing lens, 16".

Axial separation of object glass and focussing lens with the telescope in normal adjustment, 2.7".

Distance of centre of instrument from object glass, 3.7".

The centre of anallatism for near objects, say 10 feet or so from the instrument, is situated on the telescope axis close to the object glass. As the object focussed-on recedes from the level so also does the anallatic centre, and for very large object distances the anallatic constant tends asymptotically to a limiting value of about 4.9". Curve (a) indicates graphically the relation between object distances and anallatic constant, i.e. the distance of the anallatic centre from the centre of the instrument. For object distances above, say, 30 feet a fixed constant of 4.9" gives appreciably correct results, but for nearer objects it is obvious that the retention of the same constant of 4.9" will result in an over-estimate of the object distance. Curve (b) gives the percentage error introduced at different object distances on the assumption of a fixed constant of 4.9" throughout the whole range. If no constant be added at any distance it will be equally obvious that all these object distances will be under-estimated, and the corresponding percentage errors are given by curve (c). It will be noticed that, in this case, even at a distance of

100 feet, the error committed is nearly one half per cent., whilst at 10 feet the error will amount to about $3\frac{1}{2}$ per cent. It may also be of interest to observe that below about 10 feet the error begins to diminish again.



Mr T. Smith (communicated remarks) suggested a simple treatment by regarding the first principal focus of the objective and the second principal focus of the focussing lens as reference points. Instead of considering the variations in the results in absolute value he preferred treating them in proportion to the distance under measurement, and finding what the separation of the stadia lines should be and what the constant of the instrument to yield the best possible results. The quantities given in the tables did not appear to him to be comparable in the various examples, the constant κ varying throughout the series to the extent of being 50 per cent. greater in the last case than in the first. The separation between the stadia lines, on which the accuracy obtainable in use largely depends, varies in the inverse proportion to κ and is therefore 50 per cent. greater in the first case than in the last. (κ is the power when focussed for infinity.)

The author, in reply, said that Mr Shackleton's figures agreed with those in the paper as well as was to be expected; they showed that the error was practically a constant except at very short ranges.

Prof. Cheshire's suggestion did not seem practicable for the same reason that it was not practicable in the ordinary telescope.

VISUAL FATIGUE AND EYE STRAIN IN THE USE OF TELESCOPES

By R. J. E. HANSON, O.B.E., M.A., M.B.

Read and discussed, 11th November, 1920.

OCULAR fatigue may be due to various factors among which may be mentioned excessive use of the eyes for prolonged convergence and accommodation without reasonable intervals of rest.

In this paper I propose to deal shortly with:

- (A) The factor that may be denoted as mal-orientation of the eyes, head and spine.
- (B) The factor that is due to strain of the eyelids and membranous envelope of the eyeball as well as the factor of excessive contrast, in using monocular instruments, where the phase of adaptation of the open, non-aligned eye influences the function of the aligned eye.

A cursory examination of the human frame shows that the easiest posture for vision is that known as the "primary position," where the head is evenly balanced on the spine with a minimum of muscular effort and where the eyes are looking out straight ahead in a horizontal direction. Any prolonged deviation from this position causes muscular fatigue and results in visual fatigue, that is, the acuity of vision falls off. If in any instrument the method of use demands a posture on the part of the observer other than this erect posture, then the best results will only be obtained provided some support is given, say to the head, so as to reduce the muscular effort necessary to maintain the requisite attitude.

The co-ordination of eyes, head, and spine is effected partly by the "orbital cone" of muscles of the eye, partly by the "spinal polygon" of muscles between the head and spine, the latter working in a sense automatically when the former have come to the limit of their action. The macular field of view is limited to a width of some 7.5 cm. horizontally at a distance of 40 cm. from the eye*. Vertically the amount is smaller. In reading, groups of words falling in the macular field can be comprehended without movement either of the eyes in their orbit or of the head as a whole. In passing from one group of words to the next the transference is made first by the movement of the eyeballs up to a total of three such movements. If the line of type is so long that three groups will not cover it the head has to move as a whole. There is a corresponding series of motions in altitude as well as in azimuth. To attain automatic action on the part of the orbital and head muscles it is evident that a very fine co-ordinating mechanism must be at work between eyes, head, and spine, and that fatigue in any part of this muscular mechanism will throw the whole train of actions out of gear.

* It is of interest to note that at a distance of 40 cm. from the nodal point of the eye the precise proportions of the foveal and macular fields—horizontally and vertically—are as follow: fovea, H. 7.5 mm., V. 5.0 mm.; macula, H. 7.5 cm., V. 5.0 cm. I am indebted to Dr H. Hartridge, of the Physiological Research Laboratory, Cambridge, for kindly working out these dimensions.

In a monocular instrument not only must correct orientation of the head be observed, but the non-aligned eye must be protected from undue pressure from its eyelids (if closed), and from undue light contrast (if open). Undue pressure of the eyelids ("screwing up the lids") causes a temporary monocular diplopia by corneal compression adjacent to the palpebral fissure, affecting also the tension of the muscles and other tissues in the other orbit simultaneously. As the macular region of the eye has only a one-sided representation in the visual cortex of the brain it may be that the inter-retinal nerve fibres associated with the optic nerve act as a mechanism balancing the receptivity of the sensory end organs in the retina when the eyes are used in simultaneous vision of objects nearly alike; possibly even this mechanism may further stereoscopic vision. When the eyes are not so engaged, as for example when one only is aligned and the other open, the action of the inter-retinal nerve fibres may assist in dimming the relative receptivity of the non-aligned eye. The interaction of the eyes under the influence of unequal luminosity and contrast has an important influence on fatigue, for difficulty, due to this cause, in perceiving an object clearly leads to straining of the eye and emotional fatigue in addition, a condition, in fact, of "shock" in its first stage, pupil reactions assisting.

An examination of the orbits and orbital musculatures of the craniata shows how little, in the essentials, is the variation in type throughout these groups of vertebrates (fish, amphibians, reptiles, birds, and man). In the museum of the Royal College of Surgeons a whole series of illustrations of these similarities may be seen and I am indebted to the courtesy of Sir John Bland Sutton for a number of views that I was able to show on the screen at the lecture.

To sum up, visual fatigue is due partly to muscular fatigue when the postures of the body and head are strained, partly to causes specially prominent in monocular vision. To minimise the loss of acuity due to fatigue the following rules should be observed.

- (i) Avoid mal-orientation of the eyes, head, and spine (this is specially important for the student beginning to use the microscope).
- (ii) If the head must be held in position other than erect, avoid unnecessary strain to the muscles forming the "spinal polygon" by the use of a soft buffer, preferably of sponge rubber, to support the forehead.
- (iii) Protect the non-aligned eye from undue contrast and strain.
- (iv) Always take up a good "stance" either when standing or sitting, so that the body and head are held in the required positions with as little muscular effort as possible.
- (v) When using heavy instruments which have to be held in front of the eye, as telescopes, sextants, etc., have them fitted with a soft buffer against which the forehead can rest as a *point d'appui*, and let the weight of them be taken by a sling round the neck so that the weight comes directly on the spine and not on the arms.

(Specimens of such fittings for various instruments in use in the Navy were shown at the lecture.)

DISCUSSION

Captain Douglas thought Dr Hanson's work of great help to gun layers and others in eliminating fatigue and securing visual accuracy.

Instructor-Commander T. Y. Baker said that everyone using fixed instruments had experienced difficulty owing to the frequent possibility of straining the position of the head and also in closing one eye. He described and exhibited shades for sextants, theodolites, etc., which would control the position of the head and shade the unused eye.

Mr W. Shackleton said that in astronomical observation it was frequently necessary to use indirect vision as the peripheral parts of the retina are more sensitive.

Mr L. W. C. Smith said that strain in using a telescope is often due to the eye point being too close to the back lens. The curved field often causes strain through the necessity of altering the accommodation.

Major E. O. Henrici asked if the touching of a shade fixed to the theodolite during observation would not cause error.

The author, in reply, said that with regard to Mr Shackleton's remarks the amount of visual purple was greater outside the rodless fovea, and the retina was there undoubtedly more sensitive, particularly to moving objects, either actually or relatively, where luminosity conditions were low. The shades might be made very light for use in hand instruments, and in cases where the head could not be rested in them, as in the case of the theodolite, the shading of the unused eye would still be a great advantage. It was obvious that some form of sling and rod support should be employed by observers, when using a sextant, etc., in order to take the weight of the forearm and instrument on a sling suspended from the neck. The author's device for this last purpose had been adopted by the British Admiralty.

EXHIBIT OF A FOCO-COLLIMATOR

BY PROFESSOR F. J. CHESHIRE, C.B.E.

At a recent meeting of the Society Professor Cheshire exhibited some simple apparatus for testing and adjusting a foco-collimator, that is a collimator with a scale in its focal plane subtending a known angle at the principal point of the collimating lens, from which angle, and the size of the image of the scale projected by any lens system, the focal length of that system can be determined. Thus if s is the separation of the parallel lines in the collimator scale, s_1 the size of the projected image of s , f_1 the focal length of the collimator, and f the focal length of the projecting system, that is the focal length required, then:

$$f = \frac{s_1 f_1}{s} = s_1 \times \text{Const.}$$

Now this constant is usually determined by measuring s and f_1 independently. A more accurate determination can, however, be made by measuring the subtended angle directly. This obviates the necessity for knowing either the value of s or f_1 , quantities not very easily determined to the necessary order of accuracy.

The foco-collimator is adjusted and its constant determined in two operations.

(1) A graticule is taken with a pair of scale lines of an unknown but convenient distance apart. This graticule is carefully adjusted in the focal plane of the collimator lens by standing the collimator vertically upon a plane reflector and focussing the graticule with a co-axial reading microscope. A sheet of thin mica is then interposed and tilted between the object-glass of the microscope and the graticule to throw light down upon the graticule and thence to the mirror. The image of the graticule scale then appears in or near to the plane of the scale itself. The latter is adjusted up and down until the object scale and its image are seen simultaneously in focus. (2) The angle subtended by the scale division is determined practically with the same auto-collimating apparatus. The mirror however is mounted vertically on an accurately-divided horizontal table and is so adjusted initially that the left of the two line images is superposed upon the right line of the graticule. The graduated table is then rotated until the right line of the image is superposed upon the left line of the graticule. The angle through which the table is rotated to produce this displacement is the angle subtended by the two scale lines at the principal point of the collimator lens, and this angle being small its reciprocal is the number by which the length of the image of the two scale lines projected by any lens must be multiplied to obtain the focal length of that lens. This constant multiplier should be engraved upon the foco-collimator.

REVIEWS

Geometrical Investigation of the Formation of Images in Optical Instruments.

Edited by M. VON ROHR; translated by R. KANTHACK. Pp. xxiii + 612; 133 figures. (To be purchased through any bookseller or directly from H.M. Stationery Office.) Price, £2. 5s. net.

The present volume is the third optical treatise translated from the German whose appearance in English is directly traceable to the war. The first, French's translation of Steinheil and Voit's work on the telescope objective, was published by private enterprise, and the other, a translation by Emsley and Swaine of Gleichen's *Theory of Modern Optical Instruments*, like the volume now under consideration, by H.M. Stationery Office. Of the two earlier books Steinheil and Voit is nothing if not practical, and, as has been pointed out before, not entirely reliable theoretically. Gleichen's book, despite its title, is of no account as a theoretical treatise, but is of value solely for the miscellaneous descriptions of instruments the author has compiled. The present volume differs radically from both of these. It is chiefly an exposition of theoretical principles, and any practical illustrations of the application of the theory are of the slightest.

We welcome the appearance of this English edition, not only because the original is perhaps the best German book on the subject that has yet appeared, or for any direct value it may have in our own industry, but chiefly because it will enable our own public to form a fair idea of the kind and extent of development which optical theory in relation to instruments has received in Germany. The book represents solid work, free from serious error, though in parts it is heavy and even difficult of digestion. The very qualities which from our point of view are somewhat of a defect may well have played an important part, in conjunction with the many admirable achievements of the Jena workshops, in earning for Germany the pre-eminent reputation she gained in the optical world. The general contents are of a character too well known already in this country, through the books of Southall and others, to need detailed description here. It may be said at once that the translation is fully justified by the importance of the contents, and the question at once occurs as to the part the book is likely to take both as a basis for technical work and as an advanced text-book. In the view of the writer it will be less utilised in either respect than might be popularly anticipated for a variety of reasons some of which may be inferred from these notes.

A large part of the book is really very elementary, and for text-book purposes would require to be compressed. Much of it is of geometrical rather than optical interest, for the imagery there considered, as was pointed out by Maxwell, is of a type incompatible with fundamental optical laws. The convention which ascribes opposite signs to the two focal lengths of a lens is undesirable, and, in general, treatment more on the lines of Gauss is in our view preferable. The more advanced parts of the book, such as those dealing with aberrations, are regarded from the analytical viewpoint, and no attempt is made to give an insight into the conditions from which the various defects arise. The treatment based on the zero-invariant is less readily adapted to calculation than other methods published in this country. The most important factor affecting the utility of the book is that since it was written methods have been devised which are at once simpler and more general.

The general impression derived from reading the volume through is that it displays the results of patient toil with well known tools rather than the achievements of men of extraordinary ability. Herein lies much encouragement for those who would see this

country again in the foremost place for optical progress. It has been asserted that our premier position was lost, not on account of inability in our scientists, but through the shortsightedness of our manufacturers. Judging from this book it is not likely to be the fault of our Universities if our former status is not regained.

As has already been said the book is free from serious defects, but in one or two respects it could be readily improved. Insistence that the focal lines are to be normally regarded as making right angles with the principal ray of a pencil is largely the cause of the failure to recognise one of them in the simplest case which arises, that of reflection or refraction at a single spherical surface. The first half of page 81 is occupied with an example of mathematical circumlocution such as rarely finds its way into print. The discussion on aberrations would be greatly improved by reference to the nature of light, a discussion on the physical basis of the effects to be expected, and the tolerances for the permissible departures from punctual ray union to which such considerations lead. It should however be remembered that this book was intended as the first volume of a more extensive work, and such questions may well have been relegated for subsequent discussion.

Among the most useful features of the book are the indexes. That for subject-matter appears unusually complete. The index of authors is exceedingly valuable for general reference purposes, but though some additions to the original list have been made in this edition we see no reference to certain books of which mention might well have been made. In addition the various chapters contain extensive historical notes which appear unusually free from national bias. As in other recent books a list of symbols is included, but we could have wished that this had been more complete. Not only is there no key to the lettering of the diagrams, but many symbols appearing in the equations developed in the book are not included in the index or have not the whole of their various meanings given. Examination of this list at once reveals one of the most serious faults we have to find, the absence of any attempt to devise a satisfactory symbolic notation. On the one hand a multiplicity of meanings is thrust upon a single type character, and on the other, though no less than twelve different kinds of type appear to be introduced for mathematical purposes, many quite distinctive characters are not used at all. Extreme complexity is at times found in a symbol which could have been readily avoided by the adoption of one of the neglected letters. If any future edition should be required we suggest, despite the very obvious difficulties, that the revision of the entire notation should receive consideration. The claim that the original should be followed does not, in our view, justify the retention of π for a variable angle in a treatise very largely concerned with relations between angles.

In conclusion the translator has carried out his work well. On the whole the book is well printed, though here and there the mathematics is not without misprints and the diagrams in places are not very clearly drawn. The paper is satisfactory and the book, as befits a volume which will be in frequent demand for reference, is well bound.

T. S.

General and Practical Optics, 3rd and revised edition. By LIONEL LAURANCE. Pp. viii + 350; 370 figures. (London: School of Optics, Ltd., 2, Guilford Place, W.C., 1920.) Price, 21s. net.

The chief fault to be found with the majority of elementary text-books in science is that the student of an enquiring turn of mind is not sufficiently well catered for. In elementary books on optics, for example, great attention is paid to "pin and string" methods of measuring refractive index, while very little reference, if any, is made to more accurate methods which do not necessarily involve the use of very complicated apparatus. Such books would be greatly improved if they laid more stress on the actual methods

employed in technical work. At the same time the elementary aspect of the subject must not be lost sight of. Mr Laurance's book marks a distinct advance along the lines suggested, although his chapter on "Practical and Experimental Work" might with advantage be somewhat amplified in a subsequent edition. The book is primarily intended for candidates for the examination of the Worshipful Company of Spectacle Makers, but is written also as a reference book for those engaged in spectacle work, for other students of optics, and as an introduction to the study of more advanced works. In its last-named capacity it could be improved by the introduction of a number of references to original papers and books where the advanced treatments are given. The book covers a wide field of subjects in a very interesting and clear manner, and numerous worked examples are given in the text to illustrate the principles involved. A large number of diagrams are also provided; the lettering in a few of these is not quite so good as it might be. The table of wave lengths on p. 7 is not quite correct, as in the Fraunhofer system of notation the G line is not due to hydrogen; G' is the hydrogen line and has a wave length of $434 \mu\mu$. Further, the mean wave length of the sodium D lines is more nearly 589 than $590 \mu\mu$. The term "dioptral" used on p. 89 is not to be recommended in preference to "dioptric." Most of the points to which exception has been taken are, however, of minor importance. The book can be thoroughly recommended to all students of elementary optics, but particularly to those who intend to qualify in the spectacle industry.

J. S. A.

Penrose's Annual: The Process Year Book. Edited by WILLIAM GAMBLE, F.R.P.S., vol. 22 ("Victory Volume"), pp. x + 112. (Bradford: Percy Lund, Humphries & Co., Ltd., 1920.) Price, 10s. 6d. net.

The present volume is the first to appear since 1916. It represents, therefore, a review of the developments that have taken place in all kinds of process work during the period 1916-19. The book contains over thirty articles, written by experts, dealing with various problems connected with what are described as the Graphic Arts. As examples of these articles one may cite "The Copying of Photographic Prints," "Irregular Grain Screens," "The Manufacture of Sensitive Papers for Technical Purposes," "News Photography," "The Process of Photo-Zincography," and "Useful Hints for the Line Operator and Etcher." The articles are well illustrated, and the book also contains a beautiful collection of illustrations representing process work in the years 1916-19. The book is to be thoroughly recommended, not only to those who are specially interested in the technique of process work, but also to all lovers of art. The illustrations alone are worth much more than the moderate price asked for the book. The editor is to be congratulated on the admirable manner in which the book has been compiled.

J. S. A.

ABSTRACTS

"The Theory of the View Finder." By Dr F. WEIDERT, *Central-Zeitung f. Opt. u. Mech.*, 41 (1920), 213, 227.

View finders are extensively used in connection with sighting telescopes employed for artillery purposes. The first form of view finder was the French *collimateur*, which consists essentially of a rectangular block of glass, one end of which is spherical. Parallel rays falling on this surface are brought to a focus at the other end of the block, on which a mark is engraved. In order to set on a distant object an observer adjusts the position of the view finder until the object is seen to coincide with the centre of the mark, the observer's eye being placed so as to look through the block at the mark and at the same

time over the block at the object. The author describes a number of modifications of the French *collimateur* and discusses the optical theory of the various types, special attention being paid to the spherical and chromatic corrections, the position of the observer's eye, and the question of brightness of mark and object.

J. S. A.

"The Optical Properties of Thick Lenses." By Dr G. HEUSSEL, *Central-Zeitung f. Opt. u. Mech.*, 41 (1920), 292, 307.

The author discusses the properties of various types of thick lenses and shows how the properties of a given type of lens vary as the thickness is gradually altered. Descriptions are given of simple models which serve to illustrate these properties. The paper is mainly mathematical.

J. S. A.

"The Physiology of Measurement." By H. SCHULZ, *Z. f. techn. Phys.*, 1 (1920), 116, 129.

The accuracy of any visual type of measurement may be increased to a certain extent by instrumental improvements, but it is ultimately limited by the purely physiological errors of the eye. The author considers what the physiological limits to accuracy in a number of types of setting are.

Two luminous points, such as stars, can be resolved, that is, seen as separate points, only if the angle subtended by them at the eye is not less than about 50 seconds of arc. Applying this principle to the case of parallel lines on a scale, it may be shown that, in order to be able to estimate with the naked eye to $\cdot 1$ of a scale division, the length of the division must not be less than about $\cdot 6$ mm., provided that the engraved marks and the pointer are very fine. This value may be reduced by using a magnifier.

The case of measuring by means of a vernier is quite different, for, in addition to the separation of the vernier and scale divisions, the accuracy depends on the length of the marks and on the size and distribution of the retinal elements on which the image is formed. The practical limit of measuring by means of a vernier may be taken as $\cdot 05$ mm. if the observations are made with the naked eye. The accuracy of the coincidence type of setting is important in connection with coincidence range-finders. The limit of accuracy is usually taken as being about 10 seconds of arc, but v. Hofe (see Abstract in *Trans. Opt. Soc.* 21 (1920), 169) has found that with expert observers the limit may be as low as 2 seconds. Errors of estimation occur in comparing the lengths of vertical lines with those of horizontal lines and in bisecting angles. In the former case a vertical line appears to be longer than a horizontal one of the same length if it subtends an angle of less than about 35 minutes of arc at the eye; for larger angles it appears to be shorter. In the case of bisecting angles there is an error of about 1 degree.

The author investigates the degree of accuracy obtainable in photometric measurements (polarimetry and colorimetry) and finds that under the most favourable conditions it is about $\cdot 001^\circ$. In the case of interference measurements one has to take into account both the photometric sensitivity and the limit of resolution. For technical measurements of length by means of interference methods the limit of accuracy obtainable is about $\cdot 05 \mu$ ($= \cdot 00005$ mm.); in the case of microscopic measurements the limit is about $\cdot 6 \mu$ ($= \cdot 0006$ mm.).

In all types of measurement it is necessary to obtain a correct estimate of the physiological errors, because in many cases these are greater than the purely instrumental ones.

J. S. A.

"The Elimination of Errors by the Method of Least Squares and Higher Powers." By J. BIRKENBACH, *Z. f. Vermessungswesen*, 49 (1920), 497.

If N observations of any quantity are denoted by $x_1, x_2, \dots, x_N$, the probable value $\bar{x}$ of the quantity is that value for which the sum of the $2n$ th powers of the apparent errors is a minimum, that is, when

$$\sum_{k=1}^{k=N} (x_k - \bar{x})^{2n} = \min.$$

n being one of the natural numbers.

A common method of determining $\bar{x}$ is to put $n = 1$ (method of least squares). The author deals with the general case and deduces the result that the probability that an error ϵ lies between the values a and b is given by

$$w_b^a = \frac{\sqrt[n]{h}}{2\Gamma\left(\frac{2n+1}{2n}\right)} \int_a^b e^{-h\epsilon^{2n}} d\epsilon$$

where e is the base of the natural logarithms and h is a positive number. The general solution of the integral is given in the form of an infinite series.

J. S. A.

NOTICE

The Secretary of the Department of Scientific and Industrial Research has sent the following notice:

Sir John Francis Cleverton Snell, Member of Council of the Institution of Civil Engineers and Past President of the Institution of Electrical Engineers, has been appointed by an Order of Council dated the 23rd day of November 1920 to be a member of the Advisory Council to the Committee of the Privy Council for Scientific and Industrial Research.